Hertz potentials approach to the dynamical Casimir effect

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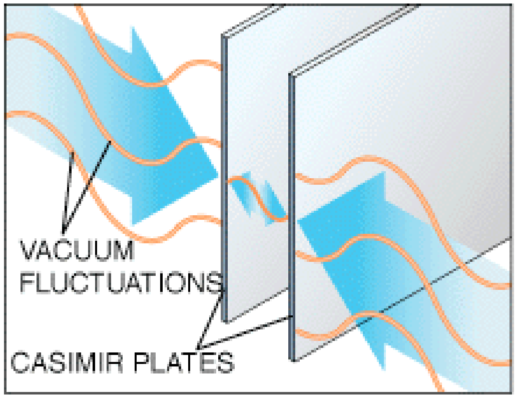
Static Casimir effect

Casimir (1948): Two uncharged, perfectly conducting plates attract each other due to the modification of the quantum vacuum fluctuations imposed by the boundaries

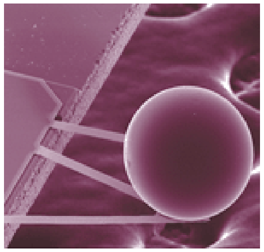


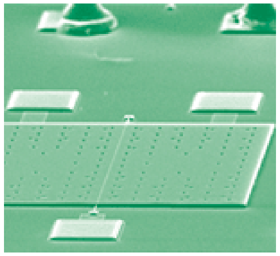
$$\boxed{\frac{F}{A} = \frac{\pi^2}{240}\frac{\hbar c}{d^4} = 0.016\frac{\text{dyn}}{\text{cm}^2}\left(\frac{\mu\text{m}}{d}\right)^4}$$

Recent experiments

❑ **Plate-sphere**

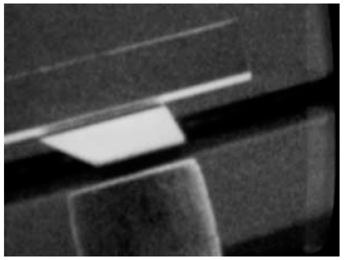
✓ **Torsion balances**
✓ **Atomic force microscopes**
✓ **Micromechanical resonators**





❑ **Parallel plates**

✓ **Cantilevers**



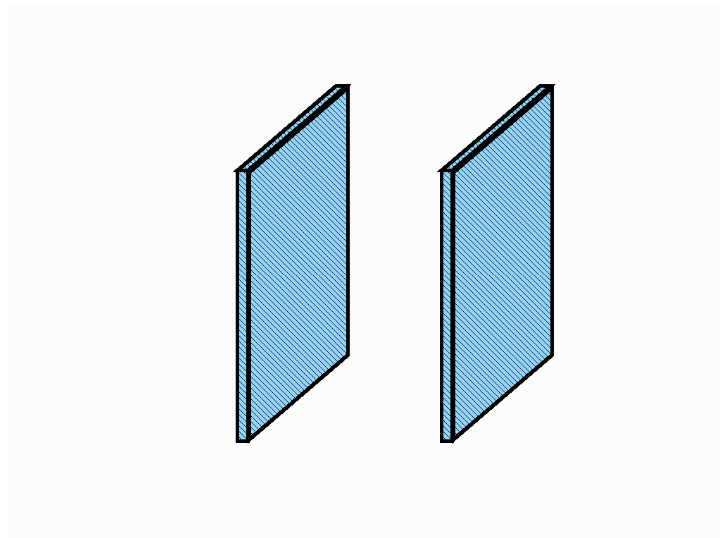
Dynamical Casimir effect

Dissipative counterpart of the conservative (static) Casimir forces

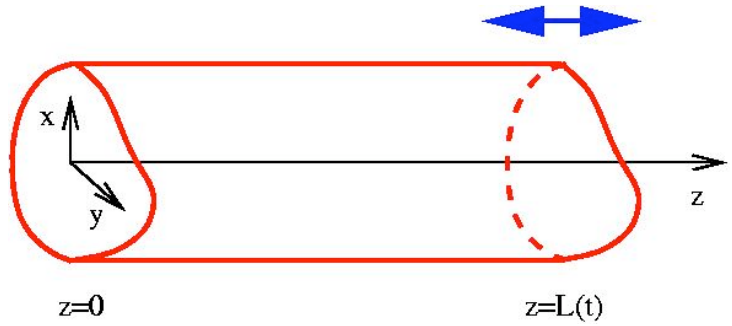
Connections and applications…

- ✓ **Non contact friction** ➡ **Vacuum-induced mechanical dissipation**

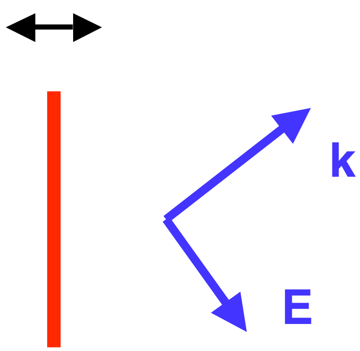
- ✓ **Motion-induced photons** ➡ **Vacuum-induced heat dissipation in the form of photons**

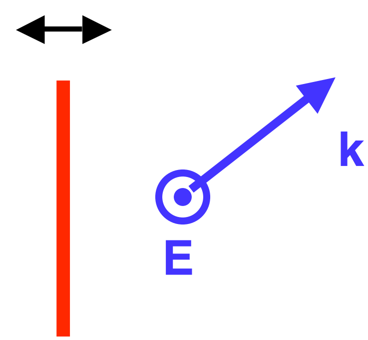


DCE in 3D waveguides (EM field)



Polarizations of the EM field:





Vector and scalar Hertz potentials

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Representation of the physical degrees of freedom of EM field alternative to the standard A and Φ

Maxwell eqns can be written in terms of two **vector potentials** Π_e **,** Π_m

(Lorentz gauge)

$$\boxed{\begin{aligned}(\mu\epsilon\,\partial_t^2 - \nabla^2)\Pi_e &= \mathbf{Q}_e \\ (\mu\epsilon\,\partial_t^2 - \nabla^2)\Pi_m &= \mathbf{Q}_m\end{aligned}}$$

[Nisbet, 1955]

$$\Phi = -\frac{1}{\epsilon}\nabla\cdot\Pi_e \qquad \rho = -\nabla\cdot\mathbf{Q}_e$$

$$\mathbf{A} = \mu\frac{\partial\Pi_e}{\partial t} + \nabla\times\Pi_m \qquad \mathbf{J} = \frac{\partial\mathbf{Q}_e}{\partial t} + \frac{1}{\mu}\nabla\times\mathbf{Q}_m$$

Stream potentials

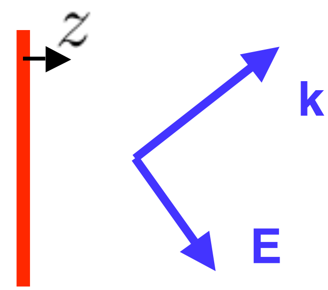
In vacuum, at points away from the sources, the electric and magnetic vector Hertz potentials can be expressed in terms of only two scalar functions, the so-called **scalar Hertz potentials** and ϕ ψ

$$\Pi_e = \phi\,\hat{\mathbf{e}}_3 \qquad \Pi_m = \psi\,\hat{\mathbf{e}}_3 \qquad (\partial_t^2 - \nabla^2)\phi = (\partial_t^2 - \nabla^2)\psi = 0$$

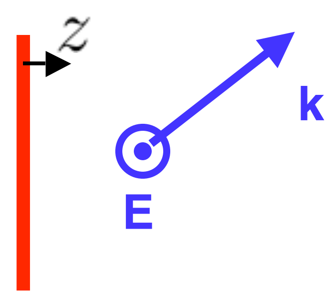
Hertz potentials (cont'd)

In previous works [eg Hacyan et al (1990); Maia Neto (1994)], the EM degrees of freedom were described using two vector potentials $\mathbf{A}_{\rm TE}$ & $\mathbf{A}_{\rm TM}$

$$\mathbf{E}_{\rm TE} = -\dot{\mathbf{A}}_{\rm TE} \ ; \ \mathbf{B}_{\rm TE} = \nabla \times \mathbf{A}_{\rm TE}$$



$$\mathbf{B}_{\rm TM} = \dot{\mathbf{A}}_{\rm TM} \ ; \ \mathbf{E}_{\rm TM} = \nabla \times \mathbf{A}_{\rm TM}$$



The two approaches are, in fact, equivalent

$$\mathbf{A}_{\rm TE} = \nabla \times \Pi_m = \hat{\mathbf{z}} \times \nabla\psi$$
$$\mathbf{A}_{\rm TM} = \nabla \times \Pi_e = \hat{\mathbf{z}} \times \nabla\phi$$

Hertz potentials (cont'd)

Boundary conditions: **starting from the usual b.c. for a perfect mirror In the instantaneous co-moving Lorentz frame**

$$\psi|_{z=0,L_z} = 0 \; ; \; \frac{\partial \psi}{\partial n}|_{\text{trans}} = 0 \qquad\qquad \frac{\partial \phi}{\partial z}|_{z=0,L_z} = 0 \; ; \; \phi|_{\text{trans}} = 0$$

TE modes

$$\boxed{\psi(z = L_z(t), t) = 0}$$

Dirichlet b.c. along z

$$\boxed{\partial_n \psi|_{\text{trans}} = 0}$$

Neumann b.c. transv.

TM modes

$$\boxed{\left(\partial_z + \dot{L}_z(t)\partial_0\right) \phi(z = L_z(t), t) = 0}$$

generalized Neumann b.c. along z

$$\boxed{\phi|_{\text{trans}} = 0}$$

Dirichlet b.c. transv.

Quantization - TE modes

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Scalar Hertz potential ψ (Dirichlet boundary conditions along z)

$$\psi(z = L_z(t), t) = 0$$

Mode expansion for t<0 :

$$\psi(\mathbf{x}, t) = \sum_{\mathbf{k}} a^{\mathrm{IN}}_{\mathbf{k}} \, u^{\mathrm{IN}}_{\mathbf{k},TE}(\mathbf{x}, t) + \mathrm{h.c.}$$

$$\mathbf{k} = (\mathbf{k}_\perp, k_z = n_z \pi / L_z)$$

$$\frac{e^{-i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}} \sqrt{\frac{2}{L_z}} \sin(k_z z) v_{\mathbf{k}_\perp}(\mathbf{x}_\perp)$$

$$\omega_{\mathbf{k}} = |\mathbf{k}|$$

The function $v_{\mathbf{k}_\perp}$ satisfies Neumann b.c. on the lateral surfaces and $\nabla^2_\perp v_{\mathbf{k}_\perp} = -\mathbf{k}^2_\perp \, v_{\mathbf{k}_\perp}$

Instantaneous basis for t >0:

$$u_{\mathbf{k},TE}(\mathbf{x}, t>0) = \sum_{\mathbf{p}} Q^{(\mathbf{k})}_{\mathbf{p},TE}(t) \sqrt{\frac{2}{L_z(t)}} \sin\left(\frac{p_z \pi}{L_z(t)} z\right) v_{\mathbf{p}_\perp}(\mathbf{x}_\perp)$$

Initial conditions: $Q^{(\mathbf{k})}_{\mathbf{p},TE}(0) = \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \delta_{\mathbf{pk}}$ $\qquad \dot{Q}^{(\mathbf{k})}_{\mathbf{p},TE}(0) = -i\sqrt{\frac{\omega_{\mathbf{k}}}{2}} \delta_{\mathbf{pk}}$

Quantization - TE modes (cont.d)

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Harmonic motion of the boundary:

$$L_z(t) = L_0[1 + \epsilon \sin(\Omega t)] \qquad \epsilon \ll 1$$

$$L_z(t < 0) = L_z(t > T) = L_0$$

Set of coupled equations for the modes:

$$\ddot{Q}^{(\mathbf{k})}_{\mathbf{p},TE} + \omega^2_{\mathbf{p}}(t) Q^{(\mathbf{k})}_{\mathbf{p},TE} = 2\lambda(t) \sum_{\mathbf{j}} g_{\mathbf{pj}} \, \dot{Q}^{(\mathbf{k})}_{\mathbf{j},TE} + \dot{\lambda}(t) \sum_{\mathbf{j}} g_{\mathbf{pj}} \, Q^{(\mathbf{k})}_{\mathbf{j},TE} + O(\epsilon^2)$$

$\dot{L}_z(t)/L_z(t)$

coupling constants

When the motion stops…

$$Q^{(\mathbf{k})}_{\mathbf{p},TE}(t > T) = A^{\mathbf{k}}_{\mathbf{p},TE} \frac{e^{-i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}} + B^{\mathbf{k}}_{\mathbf{p},TE} \frac{e^{i\omega_{\mathbf{k}} t}}{\sqrt{2\omega_{\mathbf{k}}}}$$

Motion-induced TE photons:

$$\boxed{\langle N_{\mathbf{k},TE} \rangle = \mathbf{k}^2_{\perp} \sum_{\mathbf{p}} \frac{|B^{\mathbf{k}}_{\mathbf{p},TE}|^2}{\mathbf{p}^2_{\perp}}}$$

Solving the equation for modes

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✓ **Analytical treatment: Multiple Scale Analysis (MSA)**

Parametric resonant case: $\Omega = 2\omega_{\mathbf{k}}$

A naïve perturbative solution of the mode equations in powers of $\epsilon \ll 1$ breaks down after a short amount of time, of order $(\epsilon\ \Omega)^{-1}$

MSA: ✓ **resummation of the perturbative series** $\epsilon^n\ t^n \quad (n = 1, 2, \ldots)$

✓ **solution valid for longer times,** $\epsilon^{-2}\ \Omega^{-1}$

New time scale: $\tau = \epsilon\ t$

$$Q_{\mathbf{p}}^{(\mathbf{k})}(t) = Q_{\mathbf{p}}^{(\mathbf{k})(0)}(t,\tau) + \epsilon\ Q_{\mathbf{p}}^{(\mathbf{k})(1)}(t,\tau) + O(\epsilon^2)$$

Zero order: $Q_{\mathbf{p}}^{(\mathbf{k})(0)}(t) = A_{\mathbf{p}}^{(\mathbf{k})}(\tau)\ e^{i\omega_{\mathbf{p}}t} + B_{\mathbf{p}}^{(\mathbf{k})}(\tau) e^{-i\omega_{\mathbf{p}}t}$

First order: $\partial_t^2 Q_{\mathbf{p}}^{(\mathbf{k})(1)} + \omega_{\mathbf{p}}^2 Q_{\mathbf{p}}^{(\mathbf{k})(1)} = -2\partial_{t\tau}^2 Q_{\mathbf{p}}^{(\mathbf{k})(0)} + \mathcal{F}[Q_{\mathbf{p}}^{(\mathbf{k})(0)}, \sin(\Omega t), \cos(\Omega t)]$

Solving the eqns for modes (cont'd)

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✓ **Key idea of MSA: avoid secularities by imposing that any term** $e^{\pm i\omega_{\mathbf{p}} t}$ **in the RHS of the equations cancels out.**

$$\boxed{\begin{aligned}\Omega &= 2\omega_{\mathbf{k}} \\ \Omega &= |\omega_{\mathbf{k}} \pm \omega_{\mathbf{j}}|\end{aligned}}$$

resonant conditions

$$\frac{dA_{\mathbf{k}}^{(\mathbf{n})}}{d\tau} = -\frac{\pi^2 k_z^2}{2\omega_{\mathbf{k}} L_z^2} B_{\mathbf{k}}^{(\mathbf{n})} \delta(2\omega_{\mathbf{k}} - \Omega) + \sum_{\mathbf{j}} (-\omega_{\mathbf{j}} + \frac{\Omega}{2})\, \delta(-\omega_{\mathbf{k}} - \omega_{\mathbf{j}} + \Omega) \frac{\Omega}{2\omega_{\mathbf{k}}} g_{\mathbf{kj}} B_{\mathbf{j}}^{(\mathbf{n})}$$
$$+ \sum_{\mathbf{j}} \left[(\omega_{\mathbf{j}} + \frac{\Omega}{2})\, \delta(\omega_{\mathbf{k}} - \omega_{\mathbf{j}} - \Omega) + (\omega_{\mathbf{j}} - \frac{\Omega}{2})\, \delta(\omega_{\mathbf{k}} - \omega_{\mathbf{j}} + \Omega) \right] \frac{\Omega}{2\omega_{\mathbf{k}}} g_{\mathbf{kj}} B_{\mathbf{j}}^{(\mathbf{n})}$$

Similar equation for $B_{\mathbf{k}}^{(\mathbf{n})}$

Initial conditions: $A_{\mathbf{k}}^{(\mathbf{n})}(\tau = 0) = 0$ $\quad B_{\mathbf{k}}^{(\mathbf{n})}(\tau = 0) = \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \delta_{\mathbf{nk}}$

Solving the eqns for modes (cont'd)

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- For no inter-mode coupling [$\Omega \neq |\omega_{\mathbf{k}} \pm \omega_{\mathbf{j}}|$]

$$\ddot{Q}_{\mathbf{k}}(t) + \omega_{\mathbf{k}}^2(t)\, Q_{\mathbf{k}}(t) = 0$$

(Mathiew equation)

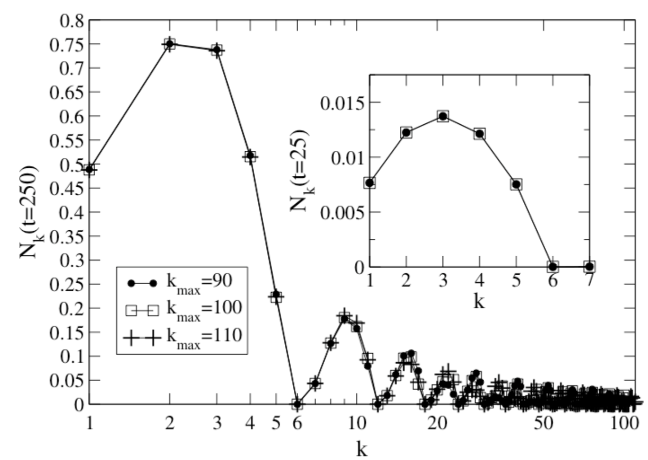
$$\boxed{N_{\mathbf{k},TE}(t) = \sinh^2(\lambda_{\mathbf{k},TE}\, \epsilon t)}$$

Exponential growth

- Similar results for finite number of coupled modes
- It is also possible to get resonant effects for small detuning

$$\Omega = 2\omega_{\mathbf{k}} + \delta \qquad \delta \simeq O(\epsilon)$$

✓ **Numerical treatment:**
see talk by Marcus Ruser (PM Today)



Quantization - TM modes

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Scalar Hertz potential ϕ (Neumann boundary conditions along z)

$$\left(\partial_z + \dot{L}_z(t)\partial_0\right)\,\phi(z = L_z(t), t) = 0$$

Mode expansion for t<0 :

$$\phi(\mathbf{x}, t) = \cancel{u_{\mathbf{k}=0}}(t) + \sum_{\mathbf{k}\neq 0} a_{\mathbf{k}}^{\mathrm{IN}}\; u_{\mathbf{k},TM}^{\mathrm{IN}}(\mathbf{x}, t) + \mathrm{h.c.}$$

Zero mode (independent of x)
It is irrelevant in a waveguide

$$\frac{e^{-i\omega_{\mathbf{k}}}}{\sqrt{2\omega_{\mathbf{k}}}}\sqrt{\frac{2}{L_z}}\cos(k_z z)\, r_{\mathbf{k}_\perp}(\mathbf{x}_\perp)$$

The function $r_{\mathbf{k}_\perp}$ satisfies Dirichlet b.c. on the lateral surfaces and $\nabla_\perp^2 r_{\mathbf{k}_\perp} = -\mathbf{k}_\perp^2 r_{\mathbf{k}_\perp}$

Instantaneous basis for t >0:

$$u_{\mathbf{k},TM} = \sum_{\mathbf{p}}\left[Q_{\mathbf{p},TM}^{(\mathbf{k})}(t) + \dot{Q}_{\mathbf{p},TM}^{(\mathbf{k})}(t)\, g(z,t)\right]\sqrt{\frac{2}{L_z(t)}}\cos\left(\frac{p_z \pi}{L_z(t)}\right) r_{\mathbf{k}_\perp}(\mathbf{x}_\perp)$$

$$\boxed{\begin{array}{ll} g(z = L_z(t), t) = 0 & \partial_z g(z = L_z(t), t) = -\dot{L}_z \\ g(z = 0, t) = 0 & \partial_z g(z = 0, t) = 0 \end{array}}$$

[talk by Mazzitelli]

Quantization - TM modes (cont'd)

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Set of coupled equations for the modes:

$$\ddot{Q}^{(\mathbf{k})}_{\mathbf{p},TM} + \omega^2_{\mathbf{k}}(t) Q^{(\mathbf{k})}_{\mathbf{p},TM} = -2\lambda(t) \sum_{\mathbf{j}} h_{\mathbf{jp}} \dot{Q}^{(\mathbf{k})}_{\mathbf{p},TM} - \dot{\lambda}(t) \sum_{\mathbf{j}} h_{\mathbf{jp}} Q^{(\mathbf{k})}_{\mathbf{p},TM}$$
$$-2\dot{\lambda}(t) L_z^2(t) \sum_{\mathbf{j}} s_{\mathbf{jp}} \ddot{Q}^{(\mathbf{k})}_{\mathbf{p},TM} - \lambda(t) L_z^2(t) \sum_{\mathbf{j}} s_{\mathbf{jp}} \partial_t^3 Q^{(\mathbf{k})}_{\mathbf{p},TM}$$
$$- \sum_{\mathbf{j}} \dot{Q}^{(\mathbf{k})}_{\mathbf{p},TM} \left[s_{\mathbf{jp}} \ddot{\lambda}(t) L_z^2(t) - \lambda(t) \eta_{\mathbf{jp}} \right] + O(\epsilon^2)$$

where $s_{\mathbf{jp}}$, $h_{\mathbf{jp}}$ and $\eta_{\mathbf{jp}}$ are coupling constants

Motion-induced TM photons:

$$\boxed{\langle N_{\mathbf{k},TM} \rangle = \mathbf{k}_\perp^2 \sum_{\mathbf{p}} \frac{|B^{\mathbf{k}}_{\mathbf{p},TM}|^2}{\mathbf{p}_\perp^2} \quad \propto \sinh^2(\lambda_{\mathbf{k},TM} \epsilon t)}$$

- ✓ Exponential growth of photons for the resonant case $\Omega = 2\omega_{\mathbf{k}}$
- ✓ Solutions independent of the particular choice of $g(z,t)$
- ✓ In general, the rate of growth for TM photons is larger than for TE photons

$$\lambda_{\mathbf{k},TE} = k_z^2 / 2\omega_{\mathbf{k}} \quad \lambda_{\mathbf{k},TM} = (2\omega_{\mathbf{k}}^2 - k_z^2)/2\omega_{\mathbf{k}} \quad \Longleftarrow \quad \boxed{\lambda_{\mathbf{k},TM} > \lambda_{\mathbf{k},TE}}$$

Cavities with rectangular section

ψ field (TE modes) : $v_{n_x,n_y}(\mathbf{x}_\perp) = \frac{2}{\sqrt{L_x L_y}} \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right)$

ϕ field (TM modes) : $r_{m_x,m_y}(\mathbf{x}_\perp) = \frac{2}{\sqrt{L_x L_y}} \sin\left(\frac{m_x \pi x}{L_x}\right) \sin\left(\frac{m_y \pi y}{L_y}\right)$

Spectrum: $\omega_{n_x,n_y,n_z} = \sqrt{(n_x\pi/L_x)^2 + (n_y\pi/L_y)^2 + (n_z\pi/Lz)^2}$

Example: resonant case $\Omega = 2\omega_{\mathbf{k}}$

✓ **TE modes** The fundamental TE mode is doubly degenerate: $(1,0,1)$ and $(0,1,1)$ and is not coupled to other modes

Growth in number of photons $\propto \exp(\pi\epsilon t/\sqrt{2}L)$

✓ **TM modes** The fundamental TM mode is $(1,1,0)$ and is coupled to $(1,1,4)$

Growth in number of photons $\propto \exp(4.4\epsilon t/L)$

$$\boxed{\omega_{TE}^{\text{fund}} = \omega_{TM}^{\text{fund}} \qquad \lambda_{TM}^{\text{fund}} > \omega_{TE}^{\text{fund}}}$$

Cavities with circular section

ψ field (TE modes) : $v_{nm}(\mathbf{x}_\perp) = \frac{1}{\sqrt{\pi}} \frac{1}{R J_n(y_{nm}) \sqrt{1 - n^2/y_{nm}^2}} J_n\left(y_{nm} \frac{\rho}{R}\right) e^{in\varphi}$

Bessel function of nth order

mth positive root of $J_n'(y) = 0$

ϕ field (TM modes) : $r_{nm}(\mathbf{x}_\perp) = \frac{1}{\sqrt{\pi}} \frac{1}{R J_{n+1}(x_{nm})} J_n\left(x_{nm} \frac{\rho}{R}\right) e^{in\varphi}$

mth root of $J_n(x) = 0$

Spectrum: $\omega_{n,m,n_z} = \sqrt{\left(\frac{y_{nm}}{R}\right)^2 + \left(\frac{n_z \pi}{L_z}\right)^2}$

Resonant case $\Omega = 2\omega_{\mathbf{k}}$

- ✓ **TE modes** The fundamental TE mode is $(1,1,1)$ and for $L_z > 2.03R$ it has a lower frequency than the fundamental TM mode. It is uncoupled

  Growth in number of photons $\propto \exp(\pi\epsilon t/\sqrt{1 + 2.912(R/L_z)^2})$

- ✓ **TM modes** The fundamental TM mode is $(0,1,0)$ and is uncoupled

  Growth in number of photons $\propto \exp(4.81\epsilon t/R)$

DCE in cavities (cont'd)

Orders of magnitude

- # of generated photons $\simeq \exp[2\lambda\epsilon t]$

  λ depends on the geometry and the particular TE or TM mode

- Amount of created photons is limited by the Q factor of the cavity

$$t_{\max} \approx Q/\omega \quad \Longrightarrow \quad N_{\max} \simeq \exp\left[\frac{2\lambda}{\omega}\,\epsilon\, Q\right]$$

- e.g. Cubic cavity:

Mode	$2\lambda/\omega$
TE (1,0,1)	0.5
TE (0,1,1)	0.5
TM (1,1,0)	1.0
TM (1,1,4)	0.3

Maximal dimensionless amplitude for mechanical oscillations $\epsilon_{\max} \approx 10^{-8}$

Very high Q factors needed to produce large number of photons!

Alternative experimental routes:

✓ MIR experiment (talks by Ruoso and Dodonov)

✓ Atom spectroscopy (talk by Onofrio)

TEM modes

Non-simply connected cavities

E and B fields do not have z components

Additional scalar field $\varphi(z,t)$

$$\mathbf{A}(\mathbf{x}_\perp, z, t) = \mathbf{A}_\perp(\mathbf{x}_\perp)\,\varphi(z,t)$$

$$\mathbf{E} = -(\partial_t \varphi)\ \mathbf{A}_\perp$$

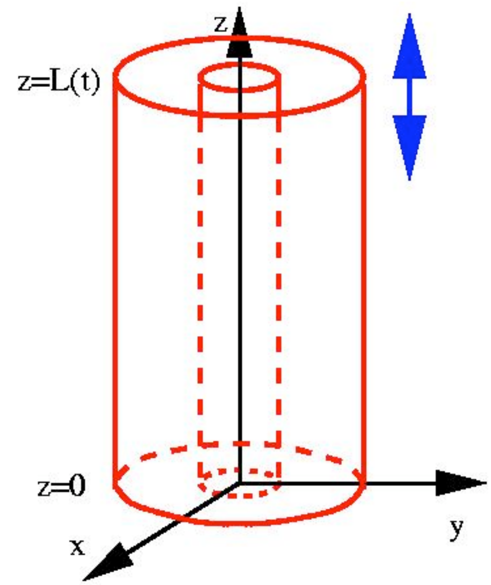
$$\mathbf{B} = (\partial_z \varphi)\ \hat{\mathbf{z}} \times \mathbf{A}_\perp$$

$$\nabla \times \mathbf{A}_\perp = \nabla \cdot \mathbf{A}_\perp = 0$$

$$\mathbf{A}_{\perp,\mathrm{tang}} = 0$$

Dirichlet b.c. along z

$$(\partial_t^2 - \partial_z^2)\varphi = 0$$



$\mathbf{A}_\perp$ s a solution of the electrostatic problem in transverse dimensions

$$H^{\mathrm{TEM}} = \frac{1}{8\pi}\left(\int d^2x_\perp |\mathbf{A}_\perp|^2\right) \int dz[(\partial_t\varphi)^2 + (\partial_z\varphi)^2]$$

Quantization of TEM modes is equivalent to quantizing a scalar field in 1+1 with Dirichlet boundary conditions.

TEM modes (cont'd)

Equidistant spectrum: $\omega_n = n\pi / L_z$ $\Rightarrow$ **all modes are coupled**

Eqns for modes can be solved as before, but it is easier (due to 1+1) to use

Moore equation [Moore 1970]:

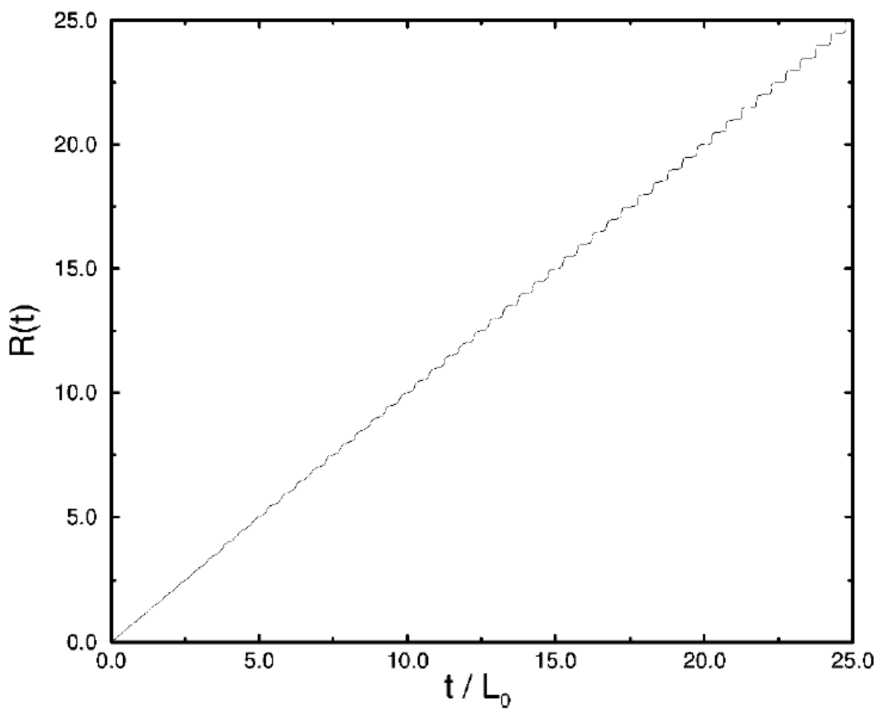
$$\psi_k(x,t) = \frac{i}{\sqrt{4\pi k}}\left(e^{-ik\pi R(t+x)} - e^{-ik\pi R(t-x)}\right)$$

$$\boxed{R(t+L(t)) - R(t-L(t)) = 2.}$$

Energy density [Fulling-Davies 1976]:

$$\langle T_{00}(x,t)\rangle = -f(t+x) - f(t-x)$$

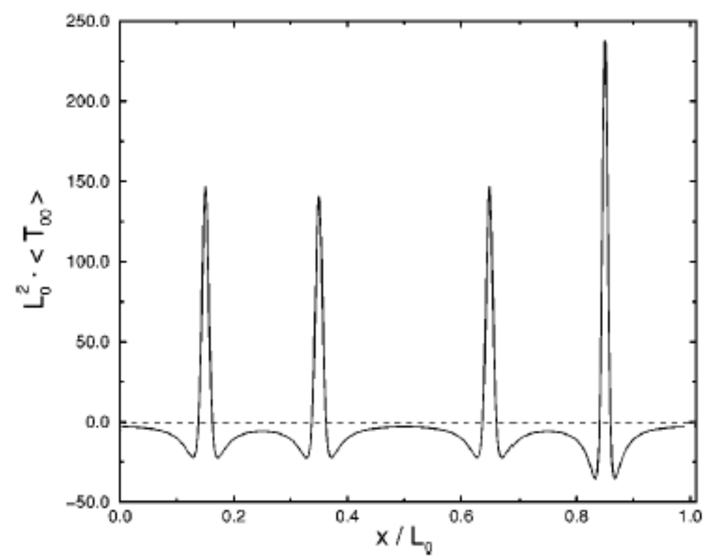
$$f = \frac{1}{24\pi}\left[\frac{R'''}{R'} - \frac{3}{2}\left(\frac{R''}{R'}\right)^2 + \frac{\pi^2}{2}(R')^2\right]$$



TEM modes (cont'd)

# of photons $\propto t^2$

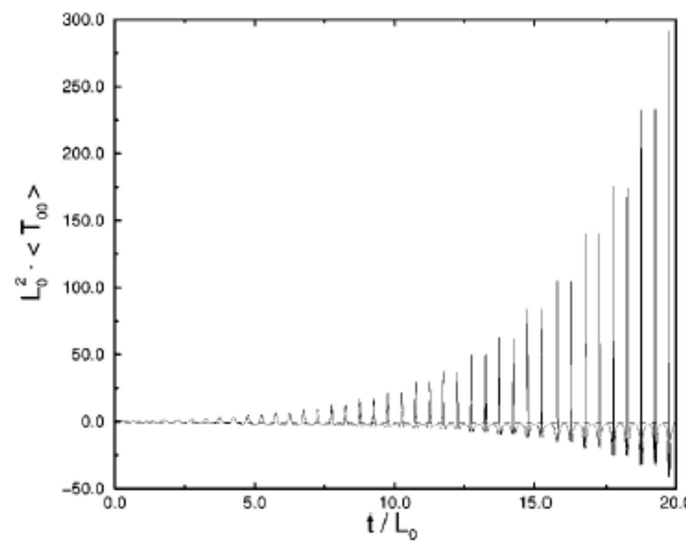
total energy $\propto \exp(\epsilon\, t)$



$\epsilon = 0.01$

$t/L_0 = 20.4$

$\Omega = 4\omega_0$



$\epsilon = 0.01$

$x/L_0 = 0.5$

$\Omega = 4\omega_0$

Conclusions

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✓ Dynamical Casimir effect with full EM field inside 3D cylindrical waveguides of arbitrary section

✓ Physical degrees of freedom of the EM field treated with scalar Hertz potentials (TE, TM, TEM modes)

✓ Coupled mode equations solved analytically using Multiple Scale Analysis (see M. Ruser's talk for numerical treatments)

References:

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